Radius of Curvature

The radius of curvature is given by

$$R \equiv \frac{1}{\kappa}, \tag{1}$$

where κ is the curvature. At a given point on a curve, R is the radius of the osculating circle. The symbol ρ is sometimes used instead of R to denote the radius of curvature.

Let x and y be given parametrically by

$$x = x(t) \tag{2}$$
$$y = y(t), \tag{3}$$

then

$$R = \frac{(x'^2 + y'^2)^{3/2}}{x'y'' - y'x''}, \tag{4}$$

where $x' = dx/dt$ and $y' = dy/dt$. Similarly, if the curve is written in the form $y = f(x)$, then the radius of curvature is given by

$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}. \tag{5}$$

In polar coordinates $r = r(\theta)$, the radius of curvature is given by

$$R = \frac{(r^2 + r_\theta^2)^{3/2}}{r^2 + 2r_\theta^2 - rr_{\theta\theta}}, \tag{6}$$

where $r_\theta = dr/d\theta$ (Gray 1997, p. 89).

SEE ALSO: Bend, Curvature, Osculating Circle, Radius of Gyration, Radius of Torsion, Torsion

References

Gray, A. *Modern Differential Geometry of Curves and Surfaces with Mathematica, 2nd ed.* Boca